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An Invitation to Quantum Game Theory

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Recent development in quantum computation and quantum information theory allows to extend the scope of game theory for the quantum world. The paper presents the history, basic ideas, and recent development in quantum game theory. In this context, a new application of the Ising chain model is proposed.

KEY WORDS: quantum games; quantum strategies; econophysics; financial markets; Ising model.

Motto

The man was very appreciative but curious. He asked the farmer why he called his horse by the wrong name three times.
The farmer said, "Oh, my horse is blind, and if he thought he was the only one pulling he wouldn't even try."

1. INTRODUCTION

Attention to the very physical aspects of information characterizes the recent research in quantum computation, quantum cryptography, and quantum communication. In most of the cases quantum description of the system provides advantages over the classical situation. For example, Simon's quantum algorithm for identifying the period of a function chosen by an oracle is more efficient than any deterministic or probabilistic algorithm (Simon, 1994); Shor's polynominal time quantum algorithm for factoring (Shor, 1994) and the quantum protocols for key distribution devised by Wiener, Bennett and Brassard, and Ekert are qualitatively more secure against eavesdropping than any classical cryptographic system (Bennett and Brassard, 1984; Ekert, 1991).

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Game theory, the study of (rational) decision making in conflict situation, seems to ask for a quantum version. For example, games against nature (Milnor, 1954) should include those for which nature is quantum mechanical. Does quantum theory present more subtle ways of playing games? Classical strategies can be pure or mixed: why cannot they be entangled? Can quantum strategies be more successful than classical ones? And if the answer is yes, are they of any practical value? Finally, von Neumann is one of the founders of both modern game theory (von Neumann and Morgenstern, 1953) and quantum theory, is that a meaningful coincidence?

2. STAR TREK: THE GAMBLING EPISODE[4]

Captain Picard and Q are characters in the popular TV series *Star Trek: The Next Generation.* Suppose they play the *spin-flip game*, which is a modern version of the penny flip game (remember that there should be no coins on a starship). Picard is to set an electron in the spin-up state, whereupon they will take turns (Q, then Picard, then Q) flipping the spin or not, without being able to see it. Q wins if the spin is up when they measure the electron's state.

This is a two-person zero-sum strategic game, which might be analyzed using the following payoff matrix:

	NN	NF	FN	FF
N	-1	1	1	-1
F	1	-1	-1	1

where the rows and columns are labeled by Picard's and Q's *pure strategies*, respectively; F denotes a flip and N denotes no flip; and the numbers in the matrix are Picard's payoffs: 1 indicating a win and -1 a loss. Q's payoffs can be obtained by reversing the signs in the above matrix (this is a general feature of a *zero-sum game*).

Example. Q's strategy is to flip the spin on his first turn and then not flip it on his second, while Picard's strategy is to not flip the spin on his turn. The result is that the state of the spin is successively U, D, D, D, and so Picard wins.

It is natural to define a two-dimensional vector space V with basis (U, D) and to represent player strategies by sequences of 2×2 matrices. That is, the matrices

$$F := \begin{array}{c c} & \begin{array}{cc} U & D \end{array} \\ \begin{array}{c} U \\ D \end{array} & \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{array} \quad \text{and} \quad N := \begin{array}{c c} & \begin{array}{cc} U & D \end{array} \\ \begin{array}{c} U \\ D \end{array} & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

[4]Based on a novel by David A. Meyer (Meyer, 1999).

correspond to flipping and not flipping the spin, respectively, since we define them to act by left multiplication on the vector representing the state of the spin. A general *mixed strategy* consists in a linear combination of F and N, which acts as a 2×2 matrix,

$$\begin{array}{c c} & \begin{array}{cc} U & D \end{array} \\ \begin{array}{c} U \\ D \end{array} & \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix} \end{array}$$

if the player flips the spin with probability $p \in [0, 1]$. A sequence of mixed actions puts the state of the electron into a convex linear combination $aU + (1-a)D$, $0 \leq a \leq 1$, which means that if the spin is measured the electron will be in the spin-up state with probability a. Q, having studied quantum theory, is utilizing a *quantum strategy*, namely a sequence of unitary, rather than stochastic, matrices. In standard Dirac notation the basis of V is written $(|U\rangle, |D\rangle)$. A *pure* quantum state for the electron is a linear combination $a|U\rangle + b|D\rangle$, $a, b \in \mathbb{C}$, $a\overline{a} + b\overline{b} = 1$, which means that if the spin is measured, the electron will be in the spin-up state with probability $a\overline{a}$. Since the electron starts in the state $|U\rangle$, this is the state of the electron if Q's first action is the unitary operation

$$U_1 = U(a, b) := \begin{array}{c c} & \begin{array}{cc} U & D \end{array} \\ \begin{array}{c} U \\ D \end{array} & \begin{pmatrix} a & b \\ \overline{b} & -\overline{a} \end{pmatrix} \end{array}$$

Captain Picard is utilizing a *classical probabilistic strategy* in which he flips the spin with probability p (he has preferred drill to studying quantum theory). After his action the electron is in a mixed quantum state, i.e., it is in the pure state $b|U\rangle + a|D\rangle$ with probability p and in the pure state $a|U\rangle + a|D\rangle$ with probability $1 - p$. Mixed states are conveniently represented as *density matrices*, elements of $V \otimes V^{\dagger}$ with trace 1; the diagonal entry (i,i) is the probability that the system is observed to be in the state $|i\rangle$. The density matrix for a pure state $|\psi\rangle \in V$ is the projection matrix $|\psi\rangle\langle\psi|$ and the density matrix for a mixed state is the corresponding convex linear combination of pure density matrices. Unitary transformations act on density matrices by conjugation: the electron starts in the pure state $\rho_0 = |U\rangle\langle U|$ and Q's first action puts it into the pure state

$$\rho_1 = U_1 \rho_0 U_1^{\dagger} = \begin{pmatrix} a\overline{a} & a\overline{b} \\ b\overline{a} & b\overline{b} \end{pmatrix}$$

Picard's mixed action acts on this density matrix, not as a stochastic matrix on a probabilistic state, but as a convex linear combination of unitary (deterministic) transformations:

$$\rho_2 = pF\rho_1 F^\dagger + (1-p)N\rho_1 N^\dagger = \begin{pmatrix} pb\overline{b} + (1-p)a\overline{a} & pb\overline{a} + (1-p)a\overline{b} \\ pa\overline{b} + (1-p)b\overline{a} & pa\overline{a} + (1-p)b\overline{b} \end{pmatrix}$$

For $p = \frac{1}{2}$, the diagonal elements of ρ_2 are equal to $\frac{1}{2}$. If the game were to end here, Picard's strategy would ensure him an expected payoff of 0, independently of Q's strategy. In fact, if Q were to employ any strategy for which $a\overline{a} \neq b\overline{b}$, Picard could obtain an expected payoff of $|a\overline{a} - b\overline{b}| > 0$ by setting $p = 0, 1$ according to whether $b\overline{b} \neq a\overline{a}$, or the reverse. Similarly if Picard were to choose $p \neq \frac{1}{2}$, Q could obtain an expected payoff of $|2p - 1|$ by setting $a = 1$ or $b = 1$ according to whether $p < \frac{1}{2}$, or the reverse. Thus the mixed/quantum equilibria for the two-move game are pairs $([\frac{1}{2}F + \frac{1}{2}N], [U(a, b)])$ for which $a\overline{a} = \frac{1}{2} = b\overline{b}$ and the outcome is the same as if both players utilize optimal mixed strategies. But Q has another move at his disposal (U_3), which again transforms the state of the electron by conjugation to $\rho_3 = U_3\rho_2 U_3^\dagger$. If Q's strategy consists of $U_1 = U(1/\sqrt{2}, 1/\sqrt{2}) = U_3$, his first action puts the electron into a simultaneous eigenvalue 1 eigenstate of both F and N, which is therefore invariant under any mixed strategy $pF + (1-p)N$ of Picard; and his second action inverts his first move to give $\rho_3 = |U\rangle\langle U|$. That is, with probability 1 the electron spin is up. Since Q can do no better than to win with probability 1, this is an optimal quantum strategy for him. All the pairs

$$([pF + (1-p)N], [U(1/\sqrt{2}, 1/\sqrt{2}), U(1/\sqrt{2}, 1/\sqrt{2})])$$

are mixed/quantum equilibria, with value -1 to Picard; this is why he loses every game. The end.

3. THE MORAL

The practical lesson that the above fable teaches is that quantum theory may offer strategies that at least in some cases bring advantage over classical strategies. Therefore game theorists should find answers to the following two questions:

- Under what conditions some players may be able to take the advantage of quantum tools?
- Are there genuine quantum games that have no classical counterparts or origin?

It is not easy to give definite answers at the present stage. Nevertheless one can present some strong arguments for developing quantum theory of games. Modern technologies are developed mostly because of investigation into the quantum nature of matter. This means that we sooner or later may wind up in captain Picard's position if we are not on alert. Secondly, quantum phenomena probably play important role in biological and other complex systems (this point of view is not commonly accepted) and quantum games may turn out to be an important tool for the analysis of complex systems. There are also suggestions that quantum-like description of market phenomena may be more accurate than the classical (probabilistic) one (Waite, 2002). The second question can be answered only after a thorough investigation. A lot of cryptographic problems can be reformulated in game-like setting. Therefore quantum information and quantum cryptography should provide us with a case in point. It is obvious that some classical games can be implemented in such a way that the set of possible strategies includes strategies that certainly deserve the adjective quantum (Du *et al.*, 2002; Pietarinen, 2002). This process is often referred to as quantization of the respective standard game. But this is an abuse of language: we are in fact defining a new game.

4. CLASSICAL GAME MAY INVOLVE QUANTUM COMPUTATION

Let us consider a game of the type of *one against all* (market). The agent buys and sells the same commodity in a consecutive way at prices dictated by the market. Let us suppose that the agent predicts with great probability the changes in price of the commodity in question. If we denote by h_m the logarithm of the relative prices $\frac{p_m}{p_{m-1}}$ at the quotation times $m = 1, 2, \ldots,$ then the total profit (loss) of the agent at the moment k is given by the formula

$$H(n_1, \ldots, n_k) := -\sum_{m=1}^{k} h_m n_m \tag{1}$$

where the series (n_m) takes the value 0 or 1 if the agent possesses money or the commodity at the moment m, respectively. Of course, the series (n_m) defines the agent's strategy in a unique way. If we take the transaction costs (e.g. brokerage) into consideration then the above formula should be replaced by

$$H(n_1, \ldots, n_k) := -\sum_{m=1}^{k} (h_m n_m - j(n_{m-1} \oplus n_m)) \tag{2}$$

where $\oplus$ denotes the addition modulo 2, $n_0 := 0$, and the constant j is equal to the logarithm of percent cost of the transaction. An attentive reader certainly notices that (2) is the Hamiltonian of an Ising chain (Feynmann, 1972) (the shift by the constant $-\frac{1}{2}$ can be absorbed into the value of h_m and therefore changes the whole formula by an unimportant constant). Classes of portfolios that correspond to the strategy $e^{-\beta H(n_1,\ldots,n_k)}$, that is to the canonical distribution, were analyzed in Piotrowski and Sladkowski (2001a). To determine the profits and correlation of agent's behavior we have to know the corresponding statistical sum, that is the logarithm of the product of the transfer matrix $M(m)$:

$$\sum_{n_1,\ldots,n_k=0}^{1} M(1)_{0,n_1} M(2)_{n_1,n_2} \cdots M(k)_{n_{k-1},n_k}$$

where

$$M(m)_{n_{m-1},n_m} := e^{\beta(h_m n_m - j(n_{m-1} \oplus n_m))}$$

Unfortunately, the matrix $M(m)$ depends on the parameter m (time) through h_m and the solution to proper value problem does not lead to a compact form of the statistical sum. It is possible to find the agent's best strategy (that is the ground state of the Hamiltonian) in the limit $\beta^{-1} \to 0^{+}$. Then the transfer matrix algebra reduces to the (min, +) algebra (Gaubert and Plus, 1997). Let us call a *potential ground state* of the Ising chain for a finite series $(h_1, \ldots, h_k)$ a strategy that, if supplemented with elements corresponding to following moments $(k' > k)$, can turn out to be the actual ground state of the Hamiltonian $H(n_1, \ldots, n_k, \ldots, n_{k'})$. These states are of the form

$$(0, 1, 1, 0, 1, 0, 0, n_{k-l+1}, n_{k-l+2}, \ldots, n_k)$$

and consist of two parts. The first one is determined by the series $(h_1, \ldots, h_k)$ and the second of length l, $(n_{k-l+2}, \ldots, n_k)$, that can be called the coherence depth (cf. the many world interpretation of quantum theory). The later can be determined only if we know h_m for $m > k$. Any potential ground state forms an optimal strategy for the agent that knows only the data up to the moment k. In this case when the transaction cost are nonzero we "discover" an obvious arbitrage risk, that for example may result from the finite maturity time of the contracts. Although the above model is classical it intrisically connected with quantum computation (and games) because all calculations for an arbitrage with nonzero transaction cost should take account of all potential ground states, number of which grows exponentially with the coherence depth. Therefore only quantum computation exploring, for example, quantum states (strategies) of the form

$$|\psi\rangle := \sum_{n_1\cdots n_k=0}^{1} c_{n_1\cdots n_k} |n_1\rangle \cdots |n_k\rangle$$

gives hope for an effective practical implementation of the strategy. This is an interesting area for further research.

5. QUANTUM GAME THEORY

Any quantum system which can be manipulated by two parties or more and where the utility of the moves can be reasonably quantified, may be conceived as a quantum game. A *two-player quantum game* $\Gamma = (\mathcal{H}, \rho, S_{\mathrm{A}}, S_{\mathrm{B}}, P_{\mathrm{A}}, P_{\mathrm{B}})$ is completely specified by the underlying Hilbert space $\mathcal{H}$ of the physical system, the initial state $\rho \in \mathcal{S}(\mathcal{H})$, where $\mathcal{S}(\mathcal{H})$ is the associated state space, the sets S_A and S_B of permissible quantum operations of the two players, and the payoff (utility) functions P_A and P_B, which specify the payoff for each player.

A *quantum strategy* $s_\mathrm{A} \in S_\mathrm{A}$, $s_\mathrm{B} \in S_\mathrm{B}$ is a quantum operation, that is, a completely positive trace-preserving map mapping the state space on itself. The quantum game's definition may also include certain additional rules, such as the order of the implementation of the respective quantum strategies. We also exclude the alteration of the payoff during the game. The generalization for the N players case is obvious.

Schematically we have

$$\rho \longmapsto (s_\mathrm{A}, s_\mathrm{B}) \longmapsto \sigma \Longrightarrow (P_\mathrm{A}, P_\mathrm{B})$$

The following concepts will be used in the remainder of this lecture. These definitions are fully analogous to the corresponding definitions in standard game theory (Osborne, 1994; Straffin, 1993). A quantum strategy s_A is called *dominant strategy* of Alice if

$$P_\mathrm{A}(s_\mathrm{A}, s'_\mathrm{B}) \geq P_\mathrm{A}(s'_\mathrm{A}, s'_\mathrm{B}) \tag{3}$$

for all $s'_\mathrm{A} \in S_\mathrm{A}$, $s'_\mathrm{B} \in S_\mathrm{B}$. Analogously we can define a dominant strategy for Bob. A pair $(s_\mathrm{A}, s_\mathrm{B})$ is said to be an *equilibrium in dominant strategies* if s_A and s_B are the players' respective dominant strategies. A combination of strategies $(s_\mathrm{A}, s_\mathrm{B})$ is called a *Nash equilibrium* if

$$P_\mathrm{A}(s_\mathrm{A}, s_\mathrm{B}) \geq P_\mathrm{A}(s'_\mathrm{A}, s_\mathrm{B}) \tag{4}$$

$$P_\mathrm{B}(s_\mathrm{A}, s_\mathrm{B}) \geq P_\mathrm{B}(s_\mathrm{A}, s'_\mathrm{B}) \tag{5}$$

A pair of strategies $(S_\mathrm{A}, S_\mathrm{B})$ is called *Pareto optimal*, if it is not possible to increase one player's payoff without lessening the payoff of the other player. A solution in dominant strategies is the strongest solution concept for a non-zero-sum game. In the Prisoner's Dilemma (Osborne, 1994; Straffin, 1993)

	Bob : C	Bob : D
Alice: C	(3, 3)	(0, 5)
Alice: D	(5, 0)	(1, 1)

the numbers in parentheses represent the row (Alice) and column (Bob) player's payoffs, respectively. Defection is the dominant strategy, as it is favorable regardless what strategy the other party chooses.

In general the optimal strategy depends on the strategy chosen by the other party. A Nash equilibrium implies that neither player has a motivation to unilaterally alter his/her strategy from this equilibrium solution, as this action will lessen his/her payoff. Given that the other player will stick to the strategy corresponding to the equilibrium, the best result is achieved by also playing the equilibrium solution. The concept of Nash equilibria is therefore of paramount importance to studies of non-zero-sum games. It is, however, only an acceptable solution concept if the Nash equilibrium is not unique. For games with multiple equilibria we have to find a way to eliminate all but one of the Nash equilibria. A Nash equilibrium is not necessarily efficient. We say that an equilibrium is Pareto optimal if there is no other outcome which would make both players better off.

Up to now several dozens of papers on quantum games have been published. We would like to mention the following important problems and proposals:

- The prescription for quantization of games provided by Eisert and coworkers (Eisert *et al.*, 1999) is a general one that can be applied to any 2×2 game, with the generalization to $2 \times n$ games. ($SU(n)$ operators are used to represent the players' actions.)
- Quantum theory of information is certainly a serious challenge to the standard game theory (e.g., quantum eavesdropping, quantum coin tossing).
- Evolutionary stable strategies (Osborne, 1994; Straffin, 1993) have been used to explaining various phenomena. Iqball and Toor (2001) have analyzed several important issues that hint that some biological systems may in fact behave in quantum-like way.
- Quantum game theory may help solving some philosophical paradoxes, cf. the quantum solution to the Newcomb's paradox (free will dilemma) (Piotrowski and Sladkowski, 2002a).
- The Monty Hall Problem (Gilman, 1992) is an interesting game based on a popular TV quiz. In this case the analysis shows that quantization of a classical game may be nonunique (Flitney and Abbott, 2002; D'Ariano *et al.*, 2002).
- In the classical Battle of Sexes Game (Osborne, 1994; Straffin, 1993) there is no satisfactory resolution. In the quantum version the deadlock may be broken (Du *et al.*, 2001). Unfortunately we see no way of using it to solve marriage problems. The analysis shows that quantization not always have direct analogies with the backgrouud classical problem.
- There are games in which the agents' strategies do not have adequate descriptions in terms of some Boolean algebra of logic and theory of probability. They can be analyzed according to the rules of quantum theory and the results are promising, see, e.g., the Wise Alice game proposed in Grib and Parfionov (2002a,b). Note that this game is a simplified version of the

Quantum Barganing Game (Piotrowski and Sladkowski, 2002b) restricted to the "quantum board" of the form [buy, sell] $\times$ [bid, accept].

- Proposals for using quantum games in market and stock exchange description (quantum market games) have already been put forward (Piotrowski and Sladkowski, 2001b, 2002b,c, in press). They seem to be very promising. At present stage, quantum auction presents a feasible idea if we neglect costs of implementation.
- Parrondo's Paradox (Harmer and Abbott, 1999) consists in asymmetrical combination of doomed games (strategies) so that the resulting new game is not biased or there even is a winning strategy. It can be used to increase reliability and stability of electrical circuits and so on. Quantum Parrondo Games are also interesting (Flitney *et al.*, 2002) and would probably find interesting applications.
- Quantum gambling: At the present stage of development it already is feasible to open "quantum casinos" (Goldenberg *et al.*, 1999; Hwang *et al.*, 2001). Quantum gambling is closely related to quantum logic and can be used to define a Bayesian theory of quantum probability (Pitowsky, 2002).
- To our knowledge, algorithmic combinatorial games, except for cellular automata, have been completely ignored by quantum physicists. This is astonishing because at least some of the important intractable problems might be attacked and solved on a quantum computer (even such a simple one player game as Minesweeper in NP-complete).

Much more can be find at, e.g., the Los Alamos preprint database.

6. SUMMARY AND OUTLOOK

We have given examples of interesting possibilities offered by quantum strategies. In general, quantum extension of a standard (classical) game is not unique. Most of the published analyses explore completely positive trace-preserving maps as admissible quantum operations (tactics or strategies). This restriction is conventional but not necessary. The effect noise and decoberence and the use of ancillas and algorithmic aspects in quantum games are the most important areas that invite further research. Quantum game theory should turn out to be an important theoretical tool for investigation of various problems in quantum cryptography and computation, economics, or game theory even if never implemented in real world. Let us quote the Editor's Note to *Complexity Digest* (2001, 27(4); http://www.comdig.org):

> It might be that while observing the due ceremonial of everyday market transaction we are in fact observing capital flows resulting from quantum games eluding classical description. If human decisions can be traced to microscopic quantum events one would expect that nature would have taken advantage of quantum computation in evolving

complex brains. In that sense one could indeed say that quantum computers are playing their market games according to quantum rules.

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